Another Undecidability Example

Let $\mathcal{L}_{101}$ be the set of encodings of TMs that accept the string 101 and no other string. Is $\mathcal{L}_{101}$ Recursively Enumerable?

Answer: No. Reduce complement of $\mathcal{L}_u$ to it.

Given (M, w) we create M'. M' takes input x. If x is 101, M' accepts x. If x is not 101 M' ignores x and simulates M on w, accepting x if M accepts w.

If M accepts w, M' accepts all strings. If M' does not accept w, M' accepts only 101.

A recognizer for $\mathcal{L}_{101}$ will recognize if M does not accept w. Thus, a recognizer for $\mathcal{L}_{101}$ creates a recognizer for the complement of $\mathcal{L}_u$, and we know that can't exist.